Reduction of execution time in the generation of dynamical planes from iterative methods

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Iterative methods give an approximate solution of equation $f(x) = 0$, $f : D \subseteq \mathbb{R}^n \longrightarrow \mathbb{R}^n$ when analytical techniques are not able to solve this problem. Root-finding method design has been a hot topic in the area of numerical analysis for the last decades.

The quality of an iterative method can be interpreted in several ways. On the one hand, the analytical quality involves the order of convergence, the amount of function evaluations, the efficiency index or the optimality, amongst others. The stability quality refers to the amount of initial guesses that converge to the expected root. Several tools, such as the convergence plane [1] or the isonormal surfaces [2], facilitates the comprehension of the dynamics of a family of methods. However, the most used representation is the dynamical plane.

The usual generation of dynamical planes iterates every single inital guess until the orbit converges to an attracting point or a maximum number of iterates is reached, as stated in [3]. Figure 1 represents the flow diagram of the process.

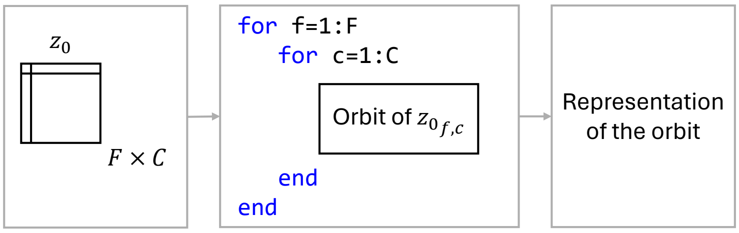

Figure 1: Flow diagram of the generation of dynamical planes

In this work, our aim is to provide a considerable improvement in the execution time of the generation of dynamical planes. The high performance code is based on a flag vector variable that controls the nodes that do not have converged yet. The results evidence the improvement in the execution time, showing that the new code uses less than a 2% of the time regarding the reference code. Some improvements can also be applied, such as reducing one half the execution time when the operators have symmetry properties [4].

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